School Choice

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Introduction

School choice is referred in the literature on market design/matching as giving parents a say in the choice of the schools their children will attend.

In some cities or countries parents have no influence in the selection of the school their children will attend (except by choosing where they live).

But in many cities school districts parents can express preferences about the schools.

School choice is another major application of matching/assignment theory.

The many-to-one assignment model

A school choice model is very close to the many-to-one matching model (e.g., the medical match). There are however some important differences.

A **school choice problem** is given by:

- A set of **students**, $I = \{i_1, \ldots, i_n\}$.
- A set of **schools**, $S = \{s_1, \ldots, s_m\}$.
- For each school $s \in S$ a **capacity**, q_s, which specifies, for each school, the maximum number of students the school can enroll.
- Each student $i \in I$ has a strict **preference ordering** P_i over the schools and the option to be unassigned.
- Each school $s \in S$ has a strict **priority ordering** π_s over the students.

An assignment problem more than a matching problem

The standard case considers **public schools**. So schools are mere objects and therefore they do not have any preferences.

This is why we assume that schools' priorities rank **all** students.

Schools' priorities are also assumed to be **responsive**.

In contrast, students may not find all schools unacceptable. Being unassigned can be viewed as:

- home schooling;
- attending a private school.

Assignments

Definition

An **assignment** is a mapping $\mu : I \cup S \to 2^I \cup S$ such that,

- $\mu(i) \in S \cup \{i\}$.
  Each student must be assigned to a school or to himself (the outside option).
- $\mu(s) \subseteq I$.
  Each school is assigned to a subset of students.
- $\mu(i) = s$ if and only if $i \in \mu(s)$.
- $|\mu(s)| \leq q_s$.

The concept of stability for school choice problems is similar to the one we used for the medical match.

Definition

An assignment is **stable** if

- it is **individually rational**: for each student $i \in I$, $\mu(i)$ is weakly preferred to the option of being unassigned.
- it is **non wasteful**: for each student $i \in I$,

$$sP_i\mu(i) \qquad \Rightarrow \qquad |\mu(s)| = q_s$$

- there is no **justified envy**: if a student i prefers a school s to his assignment then all students matched to school s must have a higher priority than student i:

$$\text{If } i, j \in I \text{ with } \mu(j) = s \in S \text{ and } sP_i\mu(i) \qquad \Rightarrow \qquad j\pi_s i$$

Efficiency

In a school choice problem, since only students have preferences welfare only takes' into account students' preferences.

Definition

An assignment μ is **efficient** if there is not other assignment μ' such that

- All students *weakly prefer* μ' to μ
  All students are either indifferent between μ' and μ or prefer μ' to μ.
- There is at least one student who strictly prefers μ' to μ
  At least one student who is not assigned to the same school under μ' and μ and prefers the school she is assigned to under μ'.

Example

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}
$s_2 s_2$	$s_1 s_1$	$s_1 s_1$	$s_2 s_2 s_2$
$s_1 s_1$	s_2	$s_2 s_2$	s_3
s_3	$s_3 s_3 s_3$	s_3	s_1

1	2	1	← capacity
π_{s_1}	π_{s_2}	π_{s_3}	
i_1	i_3	i_4	
$i_2 i_2$	i_4	i_1	
$i_3 i_3$	i_1	i_2	
i_4	i_2	i_3	

$$\mu = \{(i_1, s_1), (i_2, s_3), (i_3, s_2), (i_4, s_2)\}$$
$$\mu' = \{(i_1, s_2), (i_2, s_3), (i_3, s_1), (i_4, s_2)\}$$

Stability v. efficiency

Theorem

It may happen that, for some specific preferences and priorities, a stable assignment is also efficient.
But this is not true in general: it is impossible to guarantee to obtain at the same time efficient ***and*** *stable assignments.*

Stability and efficiency incompatible. But when they coincide, can we select the right matching?

Theorem

There is no efficient and strategy-proof mechanism that selects the efficient and stable matching whenever it exists.

The role of each side of the market

Like for assignment models, we can use algorithms like **Deferred Acceptance** or **Top Trading Cycle** (and some new ones).
All those algorithms give a precise role to each side (e.g., proposing for one side, accepting/rejecting for the other side), and two versions of the same algorithm can be obtained, depending on which side is doing what.
For school choice (or assignment problems in general), since objects/schools do not have preferences the attribution of roles is trivial:

- For DA $\rightarrow$ students propose
- For TTC $\rightarrow$ students are assigned what they point to when in cycle.

Deferred Acceptance

The Deferred Acceptance algorithm works like for the medical match:

- Students propose to schools in order of their preferences;
- Schools accept/rejects students' proposals.

The outcome of DA is the **student-optimal assignment**.
We obtain the usual results:

- DA is **strategyproof** for the students
- the student-optimal assignment is students' most preferred stable assignment.

Example

Students

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{i_5}
s_1	s_1	s_1	s_2	s_2
s_2	s_2	s_2	s_1	s_1
s_3	s_3	s_3	s_3	s_3

Schools

Schools	P_{s_1}	P_{s_2}	P_{s_3}
cap.	2	2	1
	i_1	i_5	i_1
	i_4	i_2	i_2
	i_2	i_3	i_3
	i_3	i_4	i_4
	i_5	i_1	i_5

s_1	s_2	s_3

Example

Students

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{i_5}
s_1	s_1	s_1	s_2	s_2
s_2	s_2	s_2	s_1	s_1
s_3	s_3	s_3	s_3	s_3

Schools

Schools	P_{s_1}	P_{s_2}	P_{s_3}
cap.	2	2	1
	i_1	i_5	i_1
	i_4	i_2	i_2
	i_2	i_3	i_3
	i_3	i_4	i_4
	i_5	i_1	i_5

s_1	s_2	s_3	
i_1, i_2, i_3	i_4, i_5		Students apply

Example

Students

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{i_5}
s_1	s_1	~~s_1~~	s_2	s_2
s_2	s_2	s_2	s_1	s_1
s_3	s_3	s_3	s_3	s_3

Schools

Schools	P_{s_1}	P_{s_2}	P_{s_3}
cap.	2	2	1
	i_1	i_5	i_1
	i_4	i_2	i_2
	i_2	i_3	i_3
	i_3	i_4	i_4
	i_5	i_1	i_5

s_1	s_2	s_3	
i_1, i_2, ~~i_3~~	i_4, i_5		i_3 is rejected

Example

Students

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{i_5}
s_1	s_1	~~s_1~~	s_2	s_2
s_2	s_2	s_2	s_1	s_1
s_3	s_3	s_3	s_3	s_3

Schools

Schools	P_{s_1}	P_{s_2}	P_{s_3}
cap.	2	2	1
	i_1	i_5	i_1
	i_4	i_2	i_2
	i_2	i_3	i_3
	i_3	i_4	i_4
	i_5	i_1	i_5

s_1	s_2	s_3	
i_1, i_2, ~~i_3~~	i_4, i_5		
	i_3		i_3 applies to s_2

Example

Students

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{i_5}
s_1	s_1	$\cancel{s_1}$	$\cancel{s_2}$	s_2
s_2	s_2	s_2	s_1	s_1
s_3	s_3	s_3	s_3	s_3

Schools

Schools	P_{s_1}	P_{s_2}	P_{s_3}
cap.	2	2	1
	i_1	i_5	i_1
	i_4	i_2	i_2
	i_2	i_3	i_3
	i_3	i_4	i_4
	i_5	i_1	i_5

s_1	s_2	s_3	
i_1, i_2, $\cancel{i_3}$	$\cancel{i_4}$, i_5		
	i_3		s_2 rejects i_4

Example

Students

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{i_5}
s_1	s_1	~~s_1~~	~~s_2~~	s_2
s_2	s_2	s_2	s_1	s_1
s_3	s_3	s_3	s_3	s_3

Schools

Schools	P_{s_1}	P_{s_2}	P_{s_3}
cap.	2	2	1
	i_1	i_5	i_1
	i_4	i_2	i_2
	i_2	i_3	i_3
	i_3	i_4	i_4
	i_5	i_1	i_5

s_1	s_2	s_3	
i_1, i_2, ~~i_3~~	~~i_4~~, i_5		
	i_3		
i_4			i_4 applies to s_1

Example

Students

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{i_5}
s_1	~~s_1~~	~~s_1~~	~~s_2~~	s_2
s_2	s_2	s_2	s_1	s_1
s_3	s_3	s_3	s_3	s_3

Schools

Schools	P_{s_1}	P_{s_2}	P_{s_3}
cap.	2	2	1
	i_1	i_5	i_1
	i_4	i_2	i_2
	i_2	i_3	i_3
	i_3	i_4	i_4
	i_5	i_1	i_5

s_1	s_2	s_3	
i_1, ~~i_2~~, ~~i_3~~	~~i_4~~, i_5		
	i_3		
i_4			s_1 rejects i_2

Example

Students

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{i_5}
s_1	~~s_1~~	~~s_1~~	~~s_2~~	s_2
s_2	s_2	s_2	s_1	s_1
s_3	s_3	s_3	s_3	s_3

Schools

Schools	P_{s_1}	P_{s_2}	P_{s_3}
cap.	2	2	1
	i_1	i_5	i_1
	i_4	i_2	i_2
	i_2	i_3	i_3
	i_3	i_4	i_4
	i_5	i_1	i_5

s_1	s_2	s_3	
i_1, ~~i_2~~, ~~i_3~~	~~i_4~~, i_5		
	i_3		
i_4			
	i_2		i_2 applies to s_2

Example

Students

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{i_5}
s_1	$\not{s_1}$	$\not{s_1}$	$\not{s_2}$	s_2
s_2	s_2	$\not{s_2}$	s_1	s_1
s_3	s_3	s_3	s_3	s_3

Schools

Schools	P_{s_1}	P_{s_2}	P_{s_3}
cap.	2	2	1
	i_1	i_5	i_1
	i_4	i_2	i_2
	i_2	i_3	i_3
	i_3	i_4	i_4
	i_5	i_1	i_5

s_1	s_2	s_3
i_1, $\not{i_2}$, $\not{i_3}$	$\not{i_4}$, i_5	
	$\not{i_3}$	
i_4		
	i_2	

s_2 rejects i_3

Example

Students

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{i_5}
s_1	~~s_1~~	~~s_1~~	~~s_2~~	s_2
s_2	s_2	~~s_2~~	s_1	s_1
s_3	s_3	s_3	s_3	s_3

Schools

Schools	P_{s_1}	P_{s_2}	P_{s_3}
cap.	2	2	1
	i_1	i_5	i_1
	i_4	i_2	i_2
	i_2	i_3	i_3
	i_3	i_4	i_4
	i_5	i_1	i_5

s_1	s_2	s_3	
i_1, ~~i_2~~, ~~i_3~~	~~i_4~~, i_5		
	~~i_3~~		
i_4			
	i_2		
		i_3	i_3 applies to s_3

Example

Students

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{i_5}
s_1	~~s_1~~	~~s_1~~	~~s_2~~	s_2
s_2	s_2	~~s_2~~	s_1	s_1
s_3	s_3	s_3	s_3	s_3

Schools

Schools	P_{s_1}	P_{s_2}	P_{s_3}
cap.	2	2	1
	i_1	i_5	i_1
	i_4	i_2	i_2
	i_2	i_3	i_3
	i_3	i_4	i_4
	i_5	i_1	i_5

s_1	s_2	s_3	
i_1, ~~i_2~~, ~~i_3~~	~~i_4~~, i_5		
	~~i_3~~		
i_4			
	i_2		
		i_3	no rejection

Students

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{i_5}
s_1	~~s_1~~	~~s_1~~	~~s_2~~	s_2
s_2	s_2	~~s_2~~	s_1	s_1
s_3	s_3	s_3	s_3	s_3

Schools

Schools	P_{s_1}	P_{s_2}	P_{s_3}
cap.	2	2	1
	i_1	i_5	i_1
	i_4	i_2	i_2
	i_2	i_3	i_3
	i_3	i_4	i_4
	i_5	i_1	i_5

s_1	s_2	s_3	
i_1, ~~i_2~~, ~~i_3~~	~~i_4~~, i_5		
	~~i_3~~		
i_4			
	i_2		
		i_3	
i_1, i_4	i_2, i_5	i_3	**Final assignment**

Immediate acceptance

Another algorithm (popular in practice) is the **Immediate Acceptance** (IA) algorithm (a.k.a. **Boston algorithm**).
IA is similar to DA in many aspect:

- students propose to schools in order of the preferences;
- schools accept/reject students

A key difference, however, is that schools acceptance decisions are **definitive**: once a student is accepted by a school she cannot be rejected at a later step.

The first step of the Immediate Acceptance algorithm is **identical** to the first step of the Deferred Acceptance algorithm.

- **Step 1**
  Each student applies to her most preferred, acceptable school. (if there is no such school then the student remains unassigned).
  Each school accepts students who propose to it, one by one, following the priority order, up to its capacity. The other students are rejected.

Step $\mathbf{k}, \mathbf{k} \geq \mathbf{2}$

Students rejected in the previous step apply to their most preferred, acceptable school among the schools they haven't proposed yet.
(if there is no such school the student remains unassigned).
For each school:

- Students accepted at a previous step remain accepted. The **remaining capacity** is the school's original capacity minus the number of such students.
- Accepts students who just proposed, up to the **remaining capacity** following the priority order.
  Remaining students are rejected.

End: The algorithm stops when no student is rejected or all schools have filled their capacities. Any remaining student remains unassigned.

Example

Students

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{i_5}
s_1	s_1	s_1	s_2	s_2
s_2	s_2	s_2	s_1	s_1
s_3	s_3	s_3	s_3	s_3

Schools

Schools	P_{s_1}	P_{s_2}	P_{s_3}
cap.	2	2	1
	i_1	i_5	i_1
	i_4	i_2	i_2
	i_2	i_3	i_3
	i_3	i_4	i_4
	i_5	i_1	i_5

s_1	s_2	s_3

Example

Students

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{i_5}
s_1	s_1	s_1	s_2	s_2
s_2	s_2	s_2	s_1	s_1
s_3	s_3	s_3	s_3	s_3

Schools

Schools	P_{s_1}	P_{s_2}	P_{s_3}
cap.	2	2	1
	i_1	i_5	i_1
	i_4	i_2	i_2
	i_2	i_3	i_3
	i_3	i_4	i_4
	i_5	i_1	i_5

s_1	s_2	s_3	
i_1, i_2, i_3	i_4, i_5		Students apply

Example

Students

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{i_5}
s_1	s_1	~~s_1~~	s_2	s_2
s_2	s_2	s_2	s_1	s_1
s_3	s_3	s_3	s_3	s_3

Schools

Schools	P_{s_1}	P_{s_2}	P_{s_3}
cap.	2	2	1
	i_1	i_5	i_1
	i_4	i_2	i_2
	i_2	i_3	i_3
	i_3	i_4	i_4
	i_5	i_1	i_5

s_1	s_2	s_3	
i_1, i_2, ~~i_3~~	i_4, i_5		i_3 is rejected

Example

Students

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{i_5}
s_1	s_1	~~s_1~~	s_2	s_2
s_2	s_2	s_2	s_1	s_1
s_3	s_3	s_3	s_3	s_3

Schools

Schools	P_{s_1}	P_{s_2}	P_{s_3}
cap.	2	2	1
	i_1	i_5	i_1
	i_4	i_2	i_2
	i_2	i_3	i_3
	i_3	i_4	i_4
	i_5	i_1	i_5

s_1	s_2	s_3	
i_1, i_2, ~~i_3~~	i_4, i_5		i_3 is rejected
	i_3		

Example

Students

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{i_5}
s_1	s_1	~~s_1~~	s_2	s_2
s_2	s_2	~~s_2~~	s_1	s_1
s_3	s_3	s_3	s_3	s_3

Schools

Schools	P_{s_1}	P_{s_2}	P_{s_3}
cap.	2	2	1
	i_1	i_5	i_1
	i_4	i_2	i_2
	i_2	i_3	i_3
	i_3	i_4	i_4
	i_5	i_1	i_5

s_1	s_2	s_3	
i_1, i_2, ~~i_3~~	i_4, i_5		i_3 is rejected
	~~i_3~~		s_2 full, i_3 rejected

Example

Students

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{i_5}
s_1	s_1	~~s_1~~	s_2	s_2
s_2	s_2	~~s_2~~	s_1	s_1
s_3	s_3	s_3	s_3	s_3

Schools

Schools	P_{s_1}	P_{s_2}	P_{s_3}
cap.	2	2	1
	i_1	i_5	i_1
	i_4	i_2	i_2
	i_2	i_3	i_3
	i_3	i_4	i_4
	i_5	i_1	i_5

s_1	s_2	s_3	
i_1, i_2, ~~i_3~~	i_4, i_5		i_3 is rejected
	~~i_3~~		s_2 full, i_3 rejected
		i_3	

Example

Students

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{i_5}
s_1	s_1	~~s_1~~	s_2	s_2
s_2	s_2	~~s_2~~	s_1	s_1
s_3	s_3	s_3	s_3	s_3

Schools

Schools	P_{s_1}	P_{s_2}	P_{s_3}
cap.	2	2	1
	i_1	i_5	i_1
	i_4	i_2	i_2
	i_2	i_3	i_3
	i_3	i_4	i_4
	i_5	i_1	i_5

s_1	s_2	s_3	
i_1, i_2, ~~i_3~~	i_4, i_5		i_3 is rejected
	~~i_3~~		s_2 full, i_3 rejected
		i_3	i_3 accepted

Example

Students

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{i_5}
s_1	s_1	~~s_1~~	s_2	s_2
s_2	s_2	~~s_2~~	s_1	s_1
s_3	s_3	s_3	s_3	s_3

Schools

Schools	P_{s_1}	P_{s_2}	P_{s_3}
cap.	2	2	1
	i_1	i_5	i_1
	i_4	i_2	i_2
	i_2	i_3	i_3
	i_3	i_4	i_4
	i_5	i_1	i_5

s_1	s_2	s_3	
i_1, i_2, ~~i_3~~	i_4, i_5		i_3 is rejected
	~~i_3~~		s_2 full, i_3 rejected
		i_3	no student is rejected
i_1, i_2	i_4, i_5	i_3	**Final assignment**

Step 0
For each school $s \in S$, let **the remaining capacity** be $q_s^1 = q_s$.
Step 1
Students point to their most preferred, acceptable schools(if there is none the student points to herself).
Schools point to the student with the highest priority.
A student in a cycle is assigned the school she is pointing to (or unassigned if pointing to herself) and is removed from the problem .

$$q_2^2 = \begin{cases} q_s^1 - 1 & \text{if } s \text{ is in a cycle} \\ q_s^1 & \text{if } s \text{ is in not a cycle} \end{cases}$$

Step $\mathbf{k}, \mathbf{k} \geq \mathbf{2}$

Students point to their most preferred, acceptable school **whose remaining capacity is not zero** (if there is none the student points to herself).

Schools point to the student with the highest priority among the students still present in the problem.

A student in a cycle is assigned the school she is pointing to (or unassigned if pointing to herself) and is removed from the problem .

$$q_{k+1}^2 = \begin{cases} q_k^1 - 1 & \text{if } s \text{ is in a cycle} \\ q_k^1 & \text{if } s \text{ is in not a cycle} \end{cases}$$

End

The algorithm stops when all students or all schools have been removed. Any remaining student is assigned to herself.

Example

Students

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{i_5}
$s_1 s_1$	$s_1 s_1$	$s_1 s_1$	$s_1 s_2$	$s_2 s_2$
$s_2 s_2$	$s_2 s_2$	$s_2 s_2$	$s_1 s_1$	$s_1 s_1$
$s_3 s_3$	$s_3 s_3$	$s_3 s_3$	$s_3 s_3$	$s_3 s_3$

Schools

cap.	210	210	20
	P_{s_1}	P_{s_2}	P_{s_3}
	$i_1 i_1$	$i_5 i_5$	$i_1 i_1$
	$i_4 i_4$	$i_2 i_2$	$i_2 i_2$
	$i_2 i_2$	$i_3 i_3$	$i_3 i_3$
	$i_3 i_3$	$i_4 i_4$	$i_4 i_4$
	$i_5 i_5$	$i_1 i_1$	$i_5 i_5$

Students

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{i_5}
s_1 s_1	s_1 s_1	s_1	s_2 s_2	s_2 s_2
s_2	s_2 s_2	s_2	s_1 s_1	s_1
s_3	s_3	s_3 s_3	s_3	s_3

Schools

cap.	2	2	1
	P_{s_1}	P_{s_2}	P_{s_3}
	i_1	i_5	i_1
	i_4	i_2	i_2
	i_2	i_3	i_3
	i_3	i_4	i_4
	i_5	i_1	i_5

	i_1	i_2	i_3	i_4	i_5	stable?	Efficient?
	s_1	s_2	s_3	s_1	s_2	Yes	No
IA	s_1	s_1	s_3	s_2	s_2	No	Yes
	s_1	s_1	s_3	s_2	s_2	No	Yes

Remark

In general, IA and TTC need not coincide (there might more multiple efficient assignments).

DA and TTC are **strategyproof**. What about IA?

- i_3 and s_2 block the assignment obtained with IA.
- The problem for i_3 is that by the time she asks s_2 this latter is already full.
- A better strategy for i_3 is to ask in Step 1 of IA school s_2:
    - She would "compete" with i_4 and i_5.
    - Having a higher priority than i_4 she would be accepted.

$\Rightarrow$ IA is **not** strategyproof.

The Immediate Acceptance algorithm is “popular”: it is used in many cities.

One of the attractive feature for politicians and policy makers is that it **maximizes the number of students matched to their top choice**.

But economists argue that this argument is flawed:

Parents have to be strategic and put as a first choice a school they believe they will obtain, **not the true top choice**.

At a conference organized by the Federal Reserve Bank of Chicago in 1994 ("*Midwest approaches to school reform*"), Meyer and Glazerman report:

> *It may be optimal for some families to be strategic in listing their school choices. For example, if a parent thinks that their favorite school is oversubscribed and they have a close second favorite, they may try to avoid "wasting" their first choice on a very popular school and instead list their number two school first.*

In a meeting of the *West Zone Parents Group* of the city of Boston, it was said

> *One school choice strategy is to find a school you like that is undersubscribed and put it as a top choice, or, find a school that you like that is popular and put it as a first choice and find a school that is less popular for a "safe" second choice.*

The study of school choice with assignment mechanisms started with a paper by Abdulkadiroğlu and Sönmez published in 2003.
The publication of their research was spotted by the **Boston Globe**, highlighting the flaws of the algorithm used in Boston: the **Immediate Acceptance** algorithm.
Fall 2003: Abdulkadiroğlu, Pathak, Roth and Sönmez were asked to make some recommendation for the Boston Public Schools.

Boston Public Schools (BPS):

- Over 60,000 students K–12.
- Three zones: East, West and North.
- In 2004, about
  - 4800 students entering Kindergarten
  - 4000 entering 1st grade
  - 4300 entering 6th grade
  - 4000 entering 9th grade.

Prior to 2006, the **Boston Public Schools** (PBS) used the Immediate Acceptance algorithm.

Boston is a perfect example of our school choice model: students' priorities at schools are set by the administration following specific criteria (i.e., they're not school's *preferences*):
Schools's priorities are constructed this way:

1st tier: Students with an older sibling attending the school.

2nd tier: Students living in the **walk zone** of the schools (zones are defined by the Boston Public Schools).

3rd tier: All the other students.

Then,

- 50% of a schools' seat are prioritized according to the three ties.
- 50% of a schools' seat are not prioritized

Priorities are made strict using a random draw.

Which algorithm to replace IA?

A first criterion is to use a **strategyproof** mechanism. Doing so "*levels the playing field*":

> Parents with a good understanding of IA were able to take advantage of it an game the system successfully...
> ...at the expense of the other parents.

Then, **efficiency** or **stability**? If we want

- efficiency $\rightarrow$ use TTC;
- stability $\rightarrow$ use DA (with students proposing).

Choosing between DA and TTC depends on the way we interpret schools' priorities:

- TTC implicitly assumes that students can **trade** their priorities.
- with DA (stability), priorities are **not tradable**, students have no ownership on their priorities.

At the outset the task force preferred TTC. Eventually, they settled for DA, which Boston started to use in 2007.

The New York City school match

The NYC case is different from Boston in several aspects:

- Much larger scale: 90,000+ students, 500+ different academic programs (high school).
- The source of debate was the National Resident Matching Program: officials at the NYC Department of Education wondered if it could be adapted to NYC.

School match in NYC was initially **decentralized** (creating waiting lists), and students were restricted about the number of applications they could send.

Schools in NYC are not homogeneous:

- Some schools can screen students: targeting students with specific needs and skills
  $\Rightarrow$ these schools have **preferences** over students.
- Other schools are more "classic", like in Boston.

Evidence of schools being strategic (in the decentralized procedure) convinced that school choice in NYC is a **matching problem** and not an assignment problem.
$\Rightarrow$ Deferred Acceptance is the natural choice for NYC.

Which version of DA?

Since both sides of the market are **strategic**, we have two options: the student proposing or the school proposing DA.
Choosing the student proposing version quickly appeared to be the best option:

- DA is **strategyproof** for students with the student proposing. It produces the **student-optimal matching**
- For many-to-one problems there is no mechanism that is strategyproof for the schools and that produces stable matchings.
  Here strategyproof = revealing true preferences & true capacity.

The first year of operation of DA in NYC:

- 70,000 students matched to a school on their initial choice list (an increase of 20,000 compared to previous years).
- Unmatched students are ask to submit a new preference list.
- At the end, unmatched students are matched administratively (to a school not on their choice list).
  - Previous system: 30,000 students
  - With DA: 3,000 students.

Take-away

- School choice is a **many-to-one assignment** problem. Many insights and results are the same as for the medical match model, but as an assignment problem only students' welfare matter.
- **Efficiency** and **stability** are two properties we may want. There are not compatible.
  - **Stability** can be obtained with the Deferred Acceptance (with students proposing). It is strategyproof.
  - **Efficiency** can be obtained with the Top Trading Cycle algorithm. It is strategyproof.

- The **immediate acceptance** algorithm is another possible solution, often used in practice. It produces efficient (but not stable) matchings. It is not strategyproof.
  In general, IA and TTC do not produce the same assignments.
- The city of **Boston** used IA until 2006. In 2007 it switched to DA to assign students.
  School choice in Boston is an **assignment problem**: schools do not have preferences over students.
- **New York City** switched from a decentralized to a centralized matching mechanism, using DA with students proposing.
  School choice in NYC is a **matching mechanism**: some schools are strategic and have preferences over students.